Michael Kolacki

October 19, 2015

Physics 439 with Dr. Ilie

Advanced Electromagnetic Theory

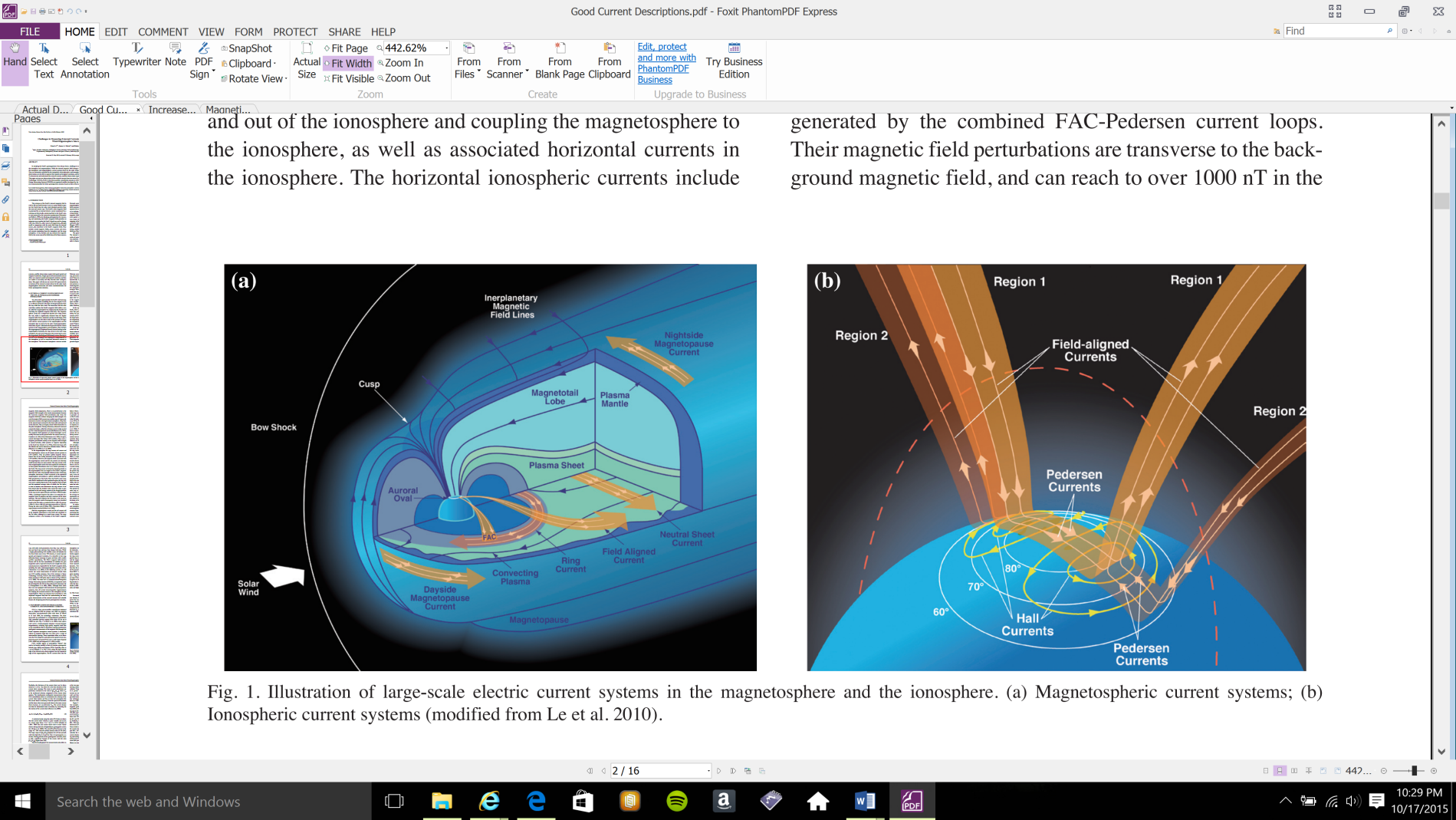
Simulations of the Magnetosphere

**Introduction:**

The Earth’s magnetosphere is an inherently complex system as it constantly interacts with and dynamically responds to an omnipresent interplanetary magnetic field and solar wind. As such, the equations governing this system are also quite complex and therefore incomprehensible to most individuals. It is for these reasons and more that accurate simulations of the system are becoming an increasingly useful and desirable tool. A description of the magnetosphere and its governing equations, as well as the simulations used to model it will be discussed in the paper that follows.

**Components of the Magnetosphere:**

There are numerous fundamental components which altogether form the magnetosphere. Some of these components include the bow shock – a supersonic wave resulting from the collision of the solar wind and magnetosphere, the heliosheath or magnetosheath – a space of higher particle density between the bow shock and magnetopause, and the magnetopause – the outer boundary of the magnetosphere [1]. Along this boundary, there is a flow of charged particles from the solar wind called the magnetopause current. The solar wind itself is a hypersonic (approximately 200 – 800 ) cloud of plasma put off by the sun’s corona which is electrically neutral, and which carries the interplanetary magnetic field (IMF), said to be “frozen in” to the plasma due to a large conductivity [2]. This means that there is no relative motion of the field to the plasma. Another important part of the magnetosphere is the magnetotail which extends beyond 50-80 (Earth Radii) from the nightside of the Earth [3]. Within the magnetotail is the plasma or neutral sheet – a sheet of plasma extending down the magnetotail and dividing the lobes into an upper and lower portion. The lobes themselves constitute the dominate portion of the magnetotail and exist between the magnetopause and neutral sheet [1]. Also within the magnetotail is the tail current, which flows in the neutral sheet from dawn to dusk. Other important currents within the magnetosphere in general include the ring current which flows westward in a circular ring around the Earth and is formed by charged particles from the solar wind being trapped in the Earth’s magnetic field, and the field aligned current (FAC) which flows in and out of the ionosphere and either into the ring current or down the magnetotail, coupling the ionosphere with the magnetosphere. The ionosphere, which is a region above the earth spanning from 100-170 km in some areas and 170-500 km in others and is partially ionized by solar radiation, contains several currents itself, namely Pedersen and Hall currents. Pedersen currents flow across the polar caps between incoming and outgoing FAC’s. Hall currents other hand are broken into two separate areas, and flow in circles around polar caps in both a westward flow on the dawn side and an eastward flow on the dusk side. These four currents are important because they contaminate geomagnetic measurements at varying degrees of intensity as they respond dynamically to changes in the solar wind and IMF. FAC’s contribute to this contamination most heavily at auroral latitudes, while the ring current is the primary source of contamination at the equatorial level. Due to the direction of the ring current’s flow and the direction of the natural magnetic field of the earth (southward through the axis and northward at the earth’s surface), the measurements of the geomagnetic field at the equator during times of solar storms is depressed. The average value of this depression is called the Dst index, and is more negative with increasingly intense solar activity. This index is used to classify geomagnetic storms, where values of more than -100 nT (nano-Tesla) are classified as moderate, values of -250 nT < Dst < -100 nT are classified as intense, and values of less than -250 nT are classified as superstorms [4]. The final major component of the magnetosphere is then the plasmasphere which is situated just above the ionosphere, capturing some of its ejected plasma [1]. These numerous components may be easily observed in Figure 1 below.

  
Figure 1: (a) This image illustrates most of the components of the magnetosphere, along with several different currents and their direction of flow. (b) Here the FAC, Pedersen currents, and Hall currents are illustrated, along with their direction of flow [4].

**Governing Equations:**

The complexity of the magnetosphere with all of its dynamically responding fields, currents, and plasmas is certainly reflected in the equations which govern the system as a whole. These equations come in two different forms; those of conservative magnetohydrodynamics (MHD) for a multispecies plasma, or of ideal conservative MHD. In either case, the equations are derived from initially applying the Boltzmann equation to several different functions, with the Boltzmann equation being defined as

(Equation 1)

where is acceleration, and is some probability distribution function with parameters (a three component spatial vector) (a three component velocity vector) and (time). The subscript simply denotes that the given function is for some plasma of species . Here and are represented as

and

respectively. In multiplying this function by another function and integrating the result over space, the generalized transport equation is obtained;

. (2)

Here, is the number density of the specified plasma and the angled brackets are used to emphasize that the average value of the function within is to be used. The further derivation of the equations of MHD are quite lengthy, and may be found fully derived by P. Kominsky [5]. After said derivation, two primary systems may be obtained, as were previously stated. While the first system is useful for considering magnetic reconnection, the implementation of such a system in global simulations becomes quite difficult for numerical reasons, and can lead to unforeseen instabilities [6-7]. It is this that occasionally leads to the second system, that of ideal conservative MHD, being utilized [8]. The difference between the two is simply an assumed infinite conductivity and a neutrality of overall volumetric charge density ( C) in ideal conservative MHD. The ideal equations explicitly stated are;

.

Here, represents volumetric density, bulk plasma velocity, pressure, magnetic field, gravity, volumetric energy density, heat transfer, and electric field. An arrow above any given variables means it should be taken as a vector. It is also worth noting that and are also often dropped from the equations for simulations because they add unnecessary complexity and contribute very little to the system comparatively [4, 8-10].

**Simulating the System:**

Even with these “simplified” equations, it is quite a daunting task to make an attempt at comprehending the functionality of the magnetosphere based on its governing equations alone. It is for this reason that accurate simulations are useful tools in observing and understanding the magnetosphere. Such a tools work to serve multiple purposes; they provide a visual representation of a system that would otherwise be incomprehensible to most individuals, they reinforce our understanding of the laws which govern such a system, they provide an easily visualized explanation for seeming anomalies in data acquired from low Earth orbit (LEO) satellites, and they work to purely expand our knowledge and understanding of the universe and the way it operates. Another extremely important use for these simulations is for predicting the severity and the outcome of varying levels of solar activity. Understanding how the Earth’s magnetic field and the current flows within the magnetosphere respond to solar storms allows for preemptive measures to be taken. These measures could include a more complete testing of satellites for potential situations they may encounter in orbit, to a rational level of security being implemented for electrical systems on the ground.

**Moving Forward:**

At the current stage, simulations of the magnetosphere provide accurate enough representations to further our understanding system’s overall functionality. However, the simplifications to the MHD equations governing the simulations make it necessary that more data be acquired and compared to them to ensure of their real world accuracy [11]. Missions have recently been taken to acquire this data, and have been quite consistent with the models resulting from simulations [4, 12].

Other possible steps to be taken are to implement the conservative MHD equations for multispecies plasma to global simulations as opposed to those of an ideal conservative system, thus accounting for a finite resistivity. Such simulations might then be able to account for things like magnetic reconnection at the magnetopause, increasing the accuracy of the models in comparison to the real world. While simulations accounting for this finite resistivity exist for small scale modeling, a global implementation has remained elusive due to its sheer complexity and the difficulty inherent to keeping the models numerically accurate. Finally, methods of calculating the values representing the simulations have also been explored recently. While traditionally, simple Cartesian grids have been used to calculate values at set points on a 2D or 3D grid, there are several models beginning to use alternative methods. One such method is that of an adaptive mesh refinement (AMR). This performs the calculations modeling the system at increasingly precise levels based on the amount of change experienced by the system. This method allows for faster compilation of code, and a remarkably precise and efficient model.

**Final Remarks:**

The Earth’s magnetosphere is an extremely complex and dynamic system constantly interacting with the interplanetary magnetic field and solar wind, and governed by the equations of magnetohydrodynamics. This system contains numerous component and current flows, which have been observed through data acquisition of orbiting satellites, and which are easily observed in computer simulations. These simulations have become increasingly useful in demonstrating the system at a global scale, and have become increasingly robust in the past few years. More data needs to be acquired and compared to the existing models to ensure their correctness, but as it stands they are known to be accurate enough to provide a good understanding of the overall functionality and dynamicity of the magnetosphere at a global level.

**Bibliography:**

1. N.A.S.A., “Magnetosphere,” <http://ccmc.gsfc.nasa.gov/educational/MagnetosphereWebPage.php>, accessed October 18, 2015.
2. Andreas Schiffler, Ph. D. thesis, University of Saskatchewan, 1996
3. David P. Stern and Mauricio Peredo, N.A.S.A, “Get a Straight Answer,” <http://www-spof.gsfc.nasa.gov/Education/FAQs3.html>, accessed October 18, 2015.
4. Guan Le, James A. Slavin, and Robert F. Pfaff, “Challenges in Measuring External Currents Driven by the Solar Wind-Magnetosphere Interaction,” J. Terr. Atmos. Ocean. Sci., Vol. 26, No. 1, 11-25, (2015)
5. P. Kominsky, “Derivation of Conservative MHD Equations.” (2006)
6. Y. Hiraki, “Auroral vortex street formed by the magnetosphere–ionosphere coupling instability,” J. Ann. Geopyys. 33, 217–224, (2015)
7. Z. Fazel and H. Ebadi, “The study of magnetic reconnection in solar spicules,” J. Astrophys Space Sci, (2014), 353:47–51, DOI 10.1007/s10509-014-2026-4
8. Andrew J. Cunningham, Adam Frank, Peggy Varni`ere, Jones, Sorin Mitran, and Thomas W. Jones, “Simulating Magnetohydrodynamical Flow with Constrained Transport and Adaptive Mesh Refinement; Algorithms & Tests of the AstroBEAR Code,” J. arXiv:0710.0424v4 [astro-ph], (2009)
9. Wang C, Guo X C, Peng Z, et al., “Magnetohydrodynamics (MHD) numerical simulations on the interaction of the solar wind with the magnetosphere: A review,” Science China: Earth Sciences, 2013, 56: 1141–1157, doi: 10.1007/s11430-013-4608-3
10. Wang J, Du A M, Zhang Y, Zhang T L, Ge Y S., “Modeling the Earth’s magnetosphere under the influence of solar wind with due northward IMF by the AMR-CESE-MHD model,” Science China: Earth Sciences, (2015), 58: 1235–1242, doi: 10.1007/s11430-015-5056-z
11. T. I. Pulkkinen, “Nonlinear solar wind – magnetosphere coupling,” J. AIP, (2010)
12. N. Yagova, B. Heilig, and E. Fedorov, “Pc2-3 geomagnetic pulsations on the ground, in the ionosphere, and in the magnetosphere: MM100, CHAMP, and THEMIS observations,” J. Ann. Geophys. 33, 117